

Determine the limit by substitution.

1) $\lim_{x \rightarrow 0} (x^2 - 5)$

2) $\lim_{x \rightarrow 2} (x^3 + 5x^2 - 7x + 1)$

3) $\lim_{x \rightarrow 0} \frac{x^3 - 6x + 8}{x - 2}$

4) $\lim_{x \rightarrow 8} \frac{x^2 + 64}{x + 8}$

5) $\lim_{x \rightarrow 5} \sqrt{x^2 + 14x + 49}$

6) $\lim_{x \rightarrow 1} (x^2 - 4x)^3$

Determine the limit algebraically, if it exists.

7) $\lim_{x \rightarrow 4} \sqrt{x - 6}$

8) $\lim_{x \rightarrow 6} \frac{x + 6}{(x - 6)^2}$

9) $\lim_{x \rightarrow 7} \frac{|7 - x|}{7 - x}$

10) $\lim_{x \rightarrow -6} \frac{x^2 - 36}{x + 6}$

11) $\lim_{x \rightarrow -10} \frac{x^2 + 15x + 50}{x + 10}$

12) $\lim_{x \rightarrow 0} \frac{x^3 + 12x^2 - 5x}{5x}$

13) $\lim_{x \rightarrow 2} \frac{x^2 + 6x - 16}{x - 2}$

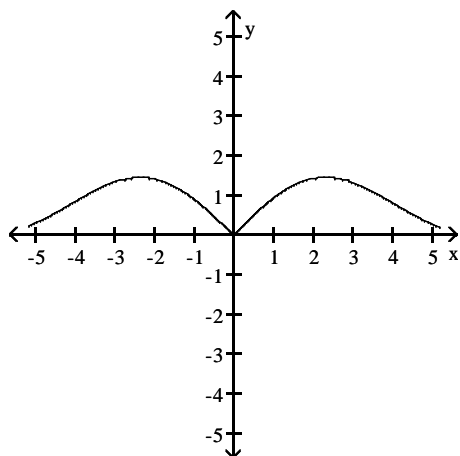
$$14) \lim_{x \rightarrow 5} \frac{x^2 + 4x - 45}{x^2 - 25}$$

$$15) \lim_{x \rightarrow 0} \frac{\frac{1}{x+8} - \frac{1}{8}}{x}$$

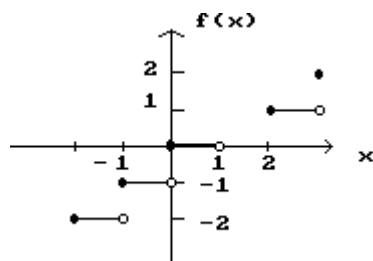
$$16) \lim_{x \rightarrow 0} \frac{10 \sin x}{6x}$$

Determine the limit graphically, if it exists.

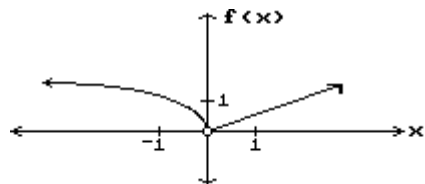
$$17) \lim_{x \rightarrow 0} f(x)$$



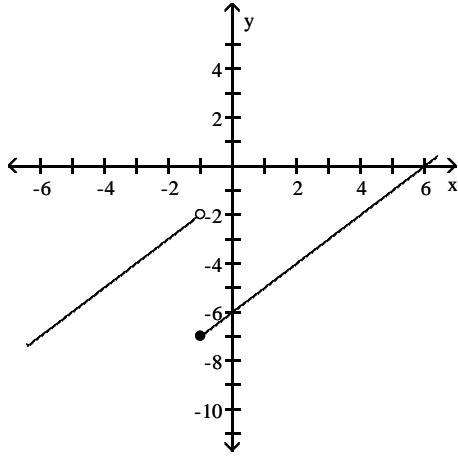
$$18) \lim_{x \rightarrow -1/2} f(x)$$



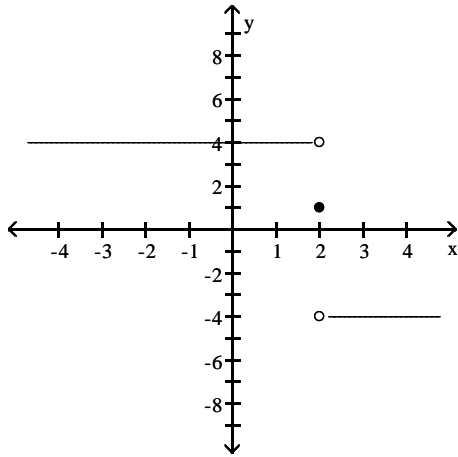
$$19) \lim_{x \rightarrow 0} f(x)$$



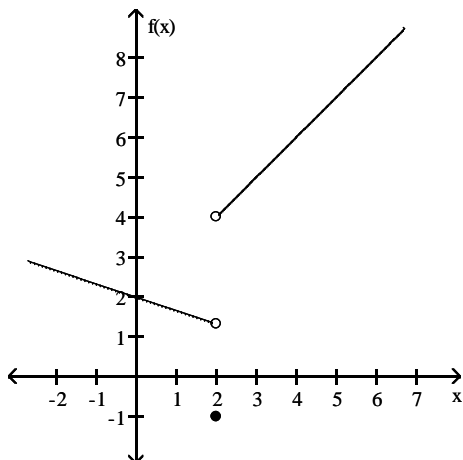
20) Find $\lim_{x \rightarrow -1^-} f(x)$ and $\lim_{x \rightarrow -1^+} f(x)$.



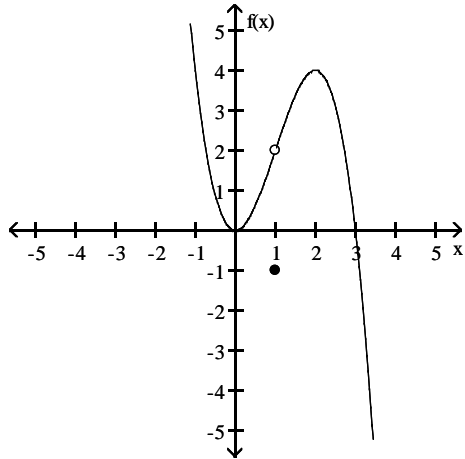
21) Find $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$.



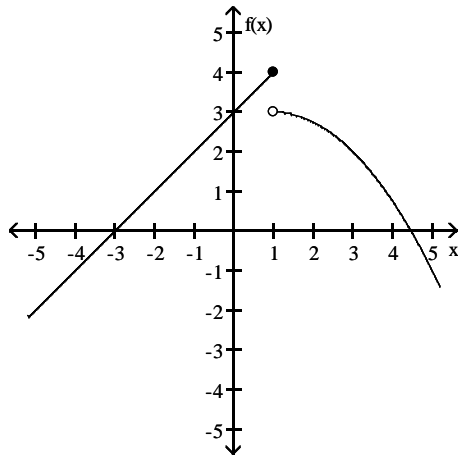
22) $\lim_{x \rightarrow 2^-} f(x)$



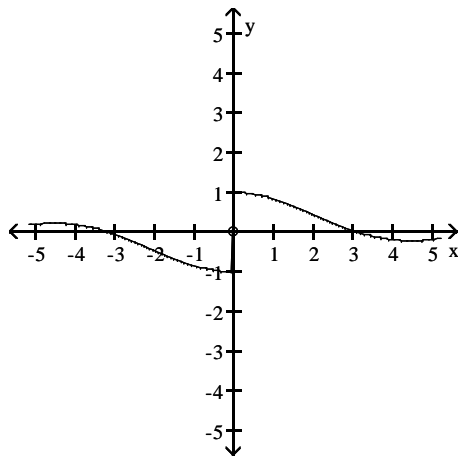
23) $\lim_{x \rightarrow 1^-} f(x)$



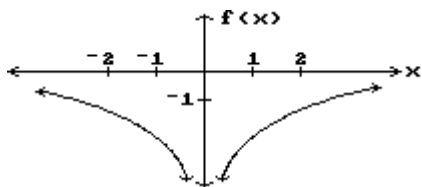
24) $\lim_{x \rightarrow 1^+} f(x)$



25) $\lim_{x \rightarrow 0} f(x)$



26) $\lim_{x \rightarrow 0} f(x)$



Find the indicated limit.

27) $\lim_{x \rightarrow 4^-} \text{int } x$

28) $\lim_{x \rightarrow -8^+} \text{int } x$

29) $\lim_{x \rightarrow 0^+} \frac{8x}{|x|}$

30) $\lim_{x \rightarrow 0^-} \frac{8x}{|x|}$

Match the function with the correct table values.

31) $f(x) = x^2 + 8x - 2$

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)						

32) $f(x) = \frac{x^4 - 1}{x - 1}$

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)						

33) $f(x) = \frac{x^3 - 6x + 8}{x - 2}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)						

34) $f(x) = \frac{x - 4}{\sqrt{x} - 2}$

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

35) $f(x) = x^2 - 5$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)						

36) $f(x) = \sqrt{x} - 2$

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

37) $f(x) = \frac{x^2 - 16}{x - 4}$

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

38) $f(x) = \frac{x + 2}{x^2 - 2x - 8}$

x	-2.1	-2.01	-2.001	-1.999	-1.99	-1.9
f(x)						

39) $f(x) = \frac{x^2 - 3x - 4}{x^2 - 2x - 8}$

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

Find the limit.

40) Let $\lim_{x \rightarrow 3} f(x) = -4$ and $\lim_{x \rightarrow 3} g(x) = 9$. Find $\lim_{x \rightarrow 3} [f(x) - g(x)]$.

41) Let $\lim_{x \rightarrow 8} f(x) = -7$ and $\lim_{x \rightarrow 8} g(x) = -6$. Find $\lim_{x \rightarrow 8} [f(x) \cdot g(x)]$.

42) Let $\lim_{x \rightarrow 9} f(x) = 10$ and $\lim_{x \rightarrow 9} g(x) = -1$. Find $\lim_{x \rightarrow 9} \frac{f(x)}{g(x)}$.

43) Let $\lim_{x \rightarrow 9} f(x) = 100$. Find $\lim_{x \rightarrow 9} \sqrt{f(x)}$.

44) Let $\lim_{x \rightarrow 5} f(x) = -4$ and $\lim_{x \rightarrow 5} g(x) = 9$. Find $\lim_{x \rightarrow 5} [f(x) + g(x)]^2$.

45) Let $\lim_{x \rightarrow -5} f(x) = -7$ and $\lim_{x \rightarrow -5} g(x) = 9$. Find $\lim_{x \rightarrow -5} \frac{-5f(x) - 9g(x)}{-5 + g(x)}$.

46) Let $\lim_{x \rightarrow -6} f(x) = -10$ and $\lim_{x \rightarrow -6} g(x) = 7$. Find $\lim_{x \rightarrow -6} \frac{[f(x)]^2}{-3 + g(x)}$.

Evaluate or determine that the limit does not exist for each of the limits (a) $\lim_{x \rightarrow d^-} f(x)$, (b) $\lim_{x \rightarrow d^+} f(x)$, and (c) $\lim_{x \rightarrow d} f(x)$

for the given function f and number d .

47)

$$f(x) = \begin{cases} x^2 - 2, & \text{for } x < 0, \\ 1, & \text{for } x \geq 0 \end{cases}$$

$$d = -4$$

48)

$$f(x) = \begin{cases} -4x - 3, & \text{for } x < 1, \\ 1, & \text{for } x = 1, \\ -2x - 3, & \text{for } x > 1 \end{cases}$$

$$d = 1$$

49)

$$f(x) = \begin{cases} -5x - 1, & \text{for } x \leq 1, \\ -7x + 1, & \text{for } x > 1 \end{cases}$$

$$d = 1$$

50)

$$f(x) = \begin{cases} \frac{1}{x-3}, & \text{for } x > 3, \\ x^2 - 4x, & \text{for } x \leq 3 \end{cases}$$

$$d = 3$$

Find the limit, if it exists.

51) $\lim_{x \rightarrow \infty} \frac{x^2 - 4x + 14}{x^3 + 9x^2 + 5}$

52) $\lim_{x \rightarrow -\infty} \frac{-8x^2 + 9x + 5}{-15x^2 + 7x + 14}$

53) $\lim_{x \rightarrow \infty} \frac{3x + 1}{11x - 7}$

54) $\lim_{x \rightarrow \infty} \frac{7x^3 - 3x^2 + 3x}{-x^3 - 2x + 7}$

$$55) \lim_{x \rightarrow -\infty} \frac{5x^3 + 4x^2}{x - 5x^2}$$

$$56) \lim_{x \rightarrow -\infty} \frac{-6 + (2/x)}{7 - (1/x^2)}$$

$$57) \lim_{x \rightarrow -\infty} \frac{\cos 3x}{x}$$

$$58) \lim_{x \rightarrow \infty} \frac{-5\sqrt{x} + x^{-1}}{-4x + 2}$$

$$59) \lim_{x \rightarrow \infty} \frac{3x^{-1} + -2x^{-3}}{3x^{-2} + x^{-5}}$$

$$60) \lim_{x \rightarrow -\infty} \frac{\sqrt[3]{x} + 5x + -5}{3x + x^{2/3} + -4}$$

Find the limit.

$$61) \lim_{x \rightarrow (-2)^-} \frac{1}{x + 2}$$

$$62) \lim_{x \rightarrow (-2)^+} \frac{1}{x + 2}$$

$$63) \lim_{x \rightarrow -2} \frac{1}{x + 2}$$

$$64) \lim_{x \rightarrow 0} \frac{1}{x^{2/3}}$$

$$65) \lim_{x \rightarrow (\pi/2)^+} \tan x$$

$$66) \lim_{x \rightarrow (-\pi/2)^-} \sec x$$

$$67) \lim_{x \rightarrow 0^+} (1 + \csc x)$$

$$68) \lim_{x \rightarrow 0} (1 - \cot x)$$

Find the vertical asymptotes of the graph of $f(x)$.

$$69) f(x) = \frac{1}{x - 8}$$

$$70) f(x) = \frac{x}{x - 9}$$

$$71) f(x) = \frac{x}{x + 8}$$

$$72) f(x) = \frac{1}{(x - 6)^2}$$

$$73) f(x) = \frac{1}{x^2 - 16}$$

$$74) f(x) = \cot x$$

$$75) f(x) = \tan x$$

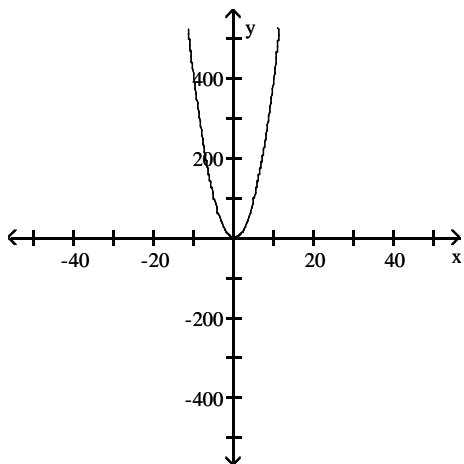
$$76) f(x) = \sec x$$

$$77) f(x) = \csc x$$

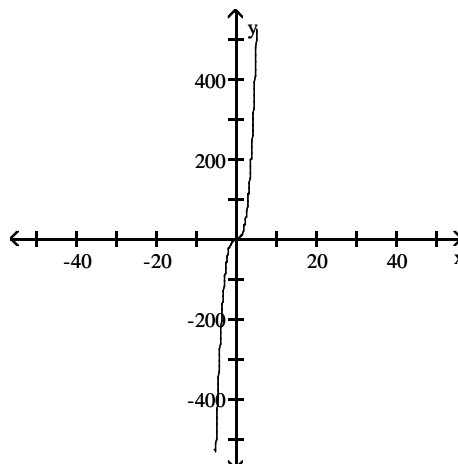
Match the function with the graph of its end behavior model.

$$78) y = \frac{4x^3 + 2x^2 + 1}{x + 5}$$

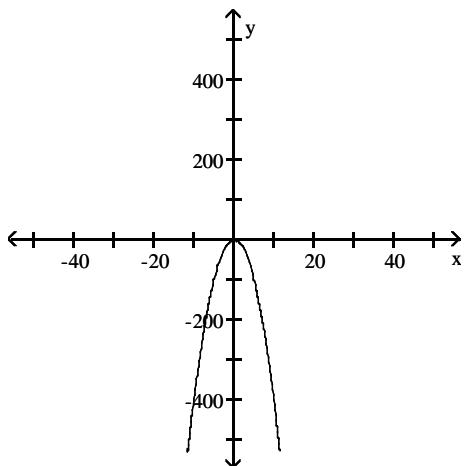
A)



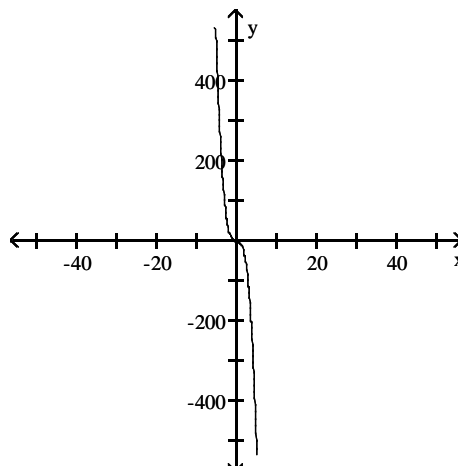
B)



C)

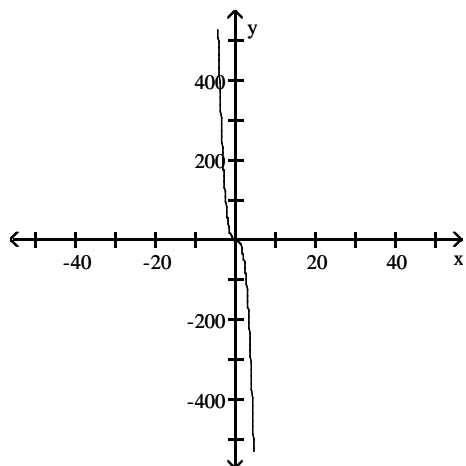


D)

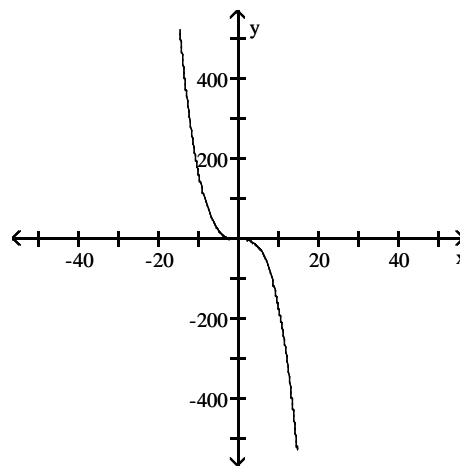


$$79) y = \frac{x^5 - x^4 + x^2 + 1}{6x^2 + 4}$$

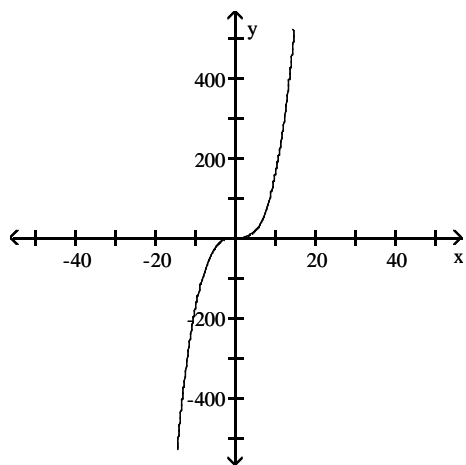
A)



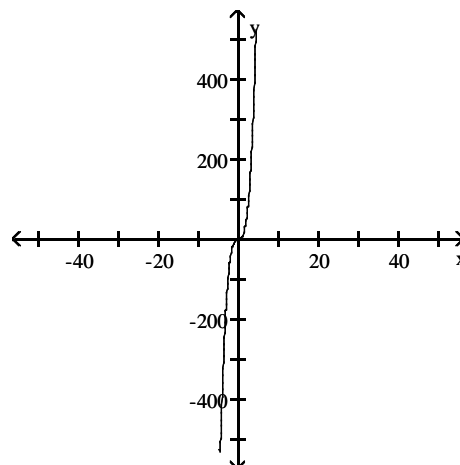
B)



C)



D)



Find a power function end behavior model.

$$80) y = 5x^2 - 7x + 2$$

$$81) y = \frac{3x^2 + x - 1}{x^3 - 8x^2}$$

Find a simple basic function as a right-end behavior model and a simple basic function as a left-end behavior model.

$$82) y = e^x + 3x$$

$$83) y = e^{-x} + 4x$$

$$84) y = \frac{x^2}{5} + e^x$$

$$85) y = \frac{x^2}{5} - e^x$$

$$86) y = 4x + \ln|x|$$

$$87) y = 7x^2 - 7 \sin x$$

$$88) y = -7x^2 - 7 \cos x$$

Find the intervals on which the function is continuous.

$$89) y = \frac{4}{(x+3)^2 + 6}$$

$$90) y = \frac{x+4}{x^2 - 12x + 32}$$

$$91) y = \sqrt{5x+1}$$

$$92) y = \sqrt[4]{10x-1}$$

$$93) y = e^{1/x}$$

$$94) y = \ln(2x-9)$$

$$95) y = \cot x$$

Find the points of discontinuity. Identify each type of discontinuity.

$$96) y = \frac{2}{(x+1)^2 + 2}$$

$$97) y = \frac{x+5}{x^2 - 12x + 35}$$

$$98) y = \sqrt{6x+3}$$

$$99) y = \sqrt[4]{6x-6}$$

$$100) y = e^{1/x}$$

$$101) y = \ln(2x-3)$$

$$102) y = \cot x$$

Find all points where the function is discontinuous.

$$103) f(x) = \begin{cases} 0, & x < 0 \\ x^2 - 7x, & 0 \leq x \leq 7 \\ 7, & x > 7 \end{cases}$$

104)

$$f(x) = \begin{cases} \frac{x^2 - 1}{x + 1}, & x \neq -1 \\ 2, & x = -1 \end{cases}$$

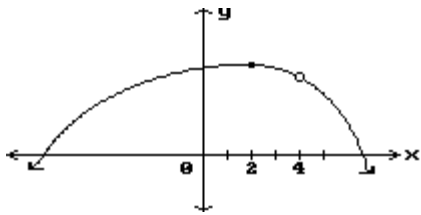
105)

$$f(x) = \begin{cases} \frac{x(x^2 - 4)}{x + 2}, & x \neq -2 \\ -3, & x = -2 \end{cases}$$

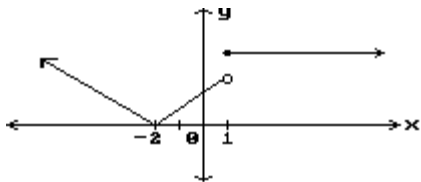
106)

$$f(x) = \begin{cases} 4, & x = 1, \\ \frac{1}{2}x^2 - 2.5, & x \neq 1; \end{cases}$$

107)

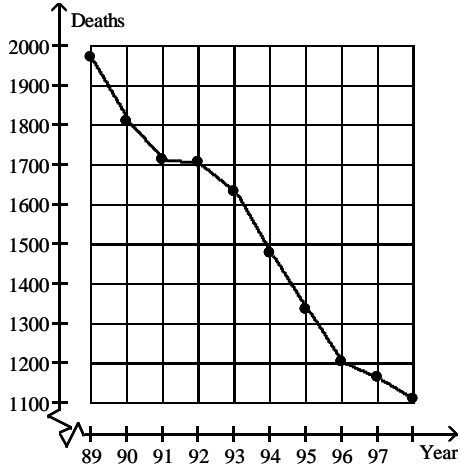


108)



Solve the problem.

109) The graph below shows the number of tuberculosis deaths in the United States from 1989 to 1998.



Estimate the average rate of change in tuberculosis deaths from 1991 to 1996.

Find the slope of the line tangent to the curve at the given value of x.

$$110) f(x) = \begin{cases} 8 + x & x \leq 6 \\ -x - 4 & x > 6 \end{cases} \quad \text{at } x = 6$$

Find the equation for the tangent to the curve at the given point.

$$111) f(x) = -8 - x^2 \quad \text{at } x = 7$$

Answer Key

Testname: CALCULUS A LIMITS AND CONTINUITY WORKSHEET

- 1) -5
- 2) 15
- 3) -4
- 4) 8
- 5) 12
- 6) -27
- 7) Does not exist
- 8) Does not exist
- 9) Does not exist
- 10) -12
- 11) -5
- 12) -1
- 13) 10
- 14) $\frac{7}{5}$
- 15) $-\frac{1}{64}$
- 16) $\frac{5}{3}$
- 17) 0
- 18) -1
- 19) 0
- 20) -2; -7
- 21) 4; -4
- 22) 1.3
- 23) 2
- 24) 3
- 25) Does not exist
- 26) Does not exist
- 27) 3
- 28) -8
- 29) 8
- 30) -8
- 31) 16.810; 17.880; 17.988; 18.012; 18.120; 19.210
- 32) 3.439; 3.940; 3.994; 4.006; 4.060; 4.641
- 33) -4.09476; -4.00995; -4.00100; -3.99900; -3.98995; -3.89526
- 34) 3.97484; 3.99750; 3.99975; 4.00025; 4.00250; 4.02485
- 35) -4.9900; -4.9999; -5.0000; -5.0000; -4.9999; -4.9900
- 36) -0.02516; -0.00250; -0.00025; 0.00025; 0.00250; 0.02485
- 37) 7.9000; 7.9900; 7.9990; 8.0010; 8.0100; 8.1000
- 38) -0.1639; -0.1664; -0.1666; -0.1667; -0.1669; -0.1695
- 39) 0.8305; 0.8331; 0.8333; 0.8334; 0.8336; 0.8361
- 40) -13
- 41) 42
- 42) -10
- 43) 10
- 44) 25
- 45) $-\frac{23}{2}$

Answer Key

Testname: CALCULUS A LIMITS AND CONTINUITY WORKSHEET

46) 25

47) (a) 14

(b) 14

(c) 14

48) (a) -7

(b) -5

(c) Does not exist

49) (a) -6

(b) -6

(c) -6

50) (a) -3

(b) Does not exist

(c) Does not exist

51) 0

52) $\frac{8}{15}$

53) $\frac{3}{11}$

54) -7

55) ∞

56) $-\frac{6}{7}$

57) 0

58) 0

59) ∞

60) $\frac{5}{3}$

61) $-\infty$

62) ∞

63) Does not exist

64) ∞

65) $-\infty$

66) ∞

67) ∞

68) Does not exist

69) $x = 8$

70) $x = 9$

71) $x = -8$

72) $x = 6$

73) $x = -4$ and $x = 4$

74) $x = n\pi$, n is any integer

75) $x = \frac{\pi}{2} + n\pi$, n is any integer

76) $x = \frac{\pi}{2} + n\pi$, n is any integer

77) $x = n\pi$, n is any integer

78) A

79) C

Answer Key

Testname: CALCULUS A LIMITS AND CONTINUITY WORKSHEET

80) $y = 5x^2$

81) $y = \frac{3}{x}$

82) $y = e^x; y = 3x$

83) $y = e^{-x}; y = 4x$

84) $y = e^x; y = \frac{x^2}{5}$

85) $y = -e^x; y = \frac{x^2}{5}$

86) $y = 4x; y = 4x$

87) $y = 7x^2; y = 7x^2$

88) $y = -7x^2; y = -7x^2$

89) $(-\infty, \infty)$

90) $(-\infty, 4), (4, 8), (8, \infty)$

91) $\left[-\frac{1}{5}, \infty\right)$

92) $\left[\frac{1}{10}, \infty\right)$

93) $(-\infty, 0), (0, \infty)$

94) $\left[\frac{9}{2}, \infty\right)$

95) $(-\infty, \infty)$ except where $x = n\pi$, n is any integer

96) None

97) $x = 5, x = 7$, both infinite discontinuities

98) $x < -\frac{1}{2}$, all points not in the domain

99) $x < 1$, all points not in the domain

100) $x = 0$, infinite discontinuity

101) $x < \frac{3}{2}$, all points not in the domain

102) $x = n\pi$, n is any integer, infinite discontinuity

103) $x = 7$

104) $x = -1$

105) $x = -2$

106) $x = 1$

107) $x = 4$

108) $x = 1$

109) About -100 deaths per year

110) Does not exist

111) $y = -14x + 41$